

# A Method for Microwave Characterization of LiNbO<sub>3</sub> Modulators

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**Abstract**—A procedure to extract the *S*-parameters of both the active section and the input and output coplanar tapers of Mach–Zehnder modulators is proposed. The *S*-parameters of the three sections of the modulator have been successfully extracted up to 30 GHz from measurements and e.m. simulations performed on proper test structures.

**Index Terms**—Coplanar tapers, coplanar waveguides, electrooptic modulators, Mach–Zehnder.

## I. INTRODUCTION

MACH-ZEHNDER interferometers are widely used in fiber-optic microwave links for electrical external modulation of optical carriers. Electrooptic substrates such as LiNbO<sub>3</sub>, covered with a low dielectric constant layer, are used to obtain velocity-matched coupling of the optical carrier with the electrical modulating signal. In the last decade, models of LiNbO<sub>3</sub> modulators have been developed in order to provide full understanding of electrooptic modulation phenomena, simulation, and optimization of optical transmitters [1]–[7].

The Mach–Zehnder modulator is composed of an active region, a coplanar waveguide (CPW) in which the electrooptic coupling comes, and an input and output structure which aims to feed and load the RF electrical field; even if the electrooptic interaction comes along the CPW only, care has to be taken in the design and modeling of the electrical launch and loading structures in order to minimize electrical mismatch and resulting back-propagation of the RF electric field. Efficiency of the coupling function requires injection of the electrical signal in a CPW with small values (about 10  $\mu\text{m}$ ) of the central conductor width, the lateral finite grounds, and the gap. The input of the launch structure and the output of the microwave load are characterized by much greater values (some hundred microns) for geometrical parameters, in order to obtain compatibility with the standard 50  $\Omega$  environment. Moreover, a 90° rotation of the electrical waveguide direction has to be ensured from the launch structure to line it to the optical waveguide. Direct electrical measurements of both the CPW and the input and output nonactive regions are precluded by their small size, not compatible with RF on-wafer measurement instrumentation. However, electrical characterization of the nonactive regions is required to develop accurate modulator models.

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In this letter, a procedure to extract the electrical *S*-parameters of both the active and non active regions of a Mach–Zehnder modulator, is proposed. The CPW is modeled by means of a transmission line, and the procedure allows to determine its attenuation and propagation constants. Then, the *S*-parameters of the input and output structures are evaluated by using both e.m. simulation, and measurements performed on proper test structures.

## II. MODEL OF THE CPW STRUCTURE

Direct extraction of CPW *S*-parameters can not be achieved, because of CPW geometrical size. Assuming a single-mode propagation regime and a matched transmission line model for the CPW, the attenuation and the propagation constants  $\alpha$  and  $\beta$  can be evaluated. Measurements performed on two structures with identical input and output tapers and different CPW lengths, allow to determine  $\alpha$  and  $\beta$  by means of the thru-reflection line (TRL) technique [5], [8]. The *S*-parameters matrix of the CPW modeled as a transmission line can be expressed in terms of  $\alpha$ ,  $\beta$ , and the reflection coefficient  $\rho$  which takes into account of the mismatch between the CPW impedance  $Z_c$  and the input/output probe tips (impedance  $Z_0$ ) [9].

The parameters  $\alpha$  and  $\beta$  can be extracted by measuring the cascade matrix of two symmetrical structures: a THRU structure containing the input and output tapers only (matrix  $A$ ) and a LINE structure containing the input taper, a  $M$ -length CPW, and the output taper (matrix  $B$ ). The cascade matrices  $A$  and  $B$  can be expressed in terms of the cascade matrices of the input taper ( $T$ ), the output taper ( $\bar{T}$ , the reverse cascade matrix of  $T$ ), and the CPW ( $T^M$ )

$$T \cdot \bar{T} = A \quad (1)$$

$$T \cdot T^M \cdot \bar{T} = B. \quad (2)$$

The values of  $\alpha$  and  $\beta$  are found by substituting into (1) and (2) the expressions of CPW *S*-parameters, and then taking and summing the determinants of (1) and (2)

$$\alpha = \left| \operatorname{Re} \left[ \frac{\operatorname{Log} (0.5 \cdot P \pm 0.5 \cdot \sqrt{-4 + P^2})}{M} \right] \right| \quad (3)$$

$$\beta = \left| \operatorname{Im} \left[ \frac{\operatorname{Log} (0.5 \cdot P \pm 0.5 \cdot \sqrt{-4 + P^2})}{M} \right] \right| \quad (4)$$

$$P = \det(A + B) - 2 \quad (5)$$

where an unwrapping algorithm is used to remove the phase jumps from the function  $\beta \cdot M$ , so making  $\beta$  a continuous function of the frequency.

It has to be noted that the expressions for  $\alpha$  and  $\beta$  are not dependent on  $Z_c$ . If the  $S$ -parameters of a  $M'$ -length CPW have to be evaluated, the characteristic impedance  $Z_c$  has to be computed from the complex propagation constant and the line specific capacitance, as discussed in [5].

### III. $S$ -PARAMETERS OF THE NON-ACTIVE REGIONS

The  $S$ -parameters of an input/output taper (nonactive regions) used to feed/load the CPW where the electrooptic modulation takes place, cannot be determined by direct measurements as discussed in Section I. A large variety of structures could be chosen and, therefore, a procedure to identify  $S$ -parameters of each of them would permit to compare their individual performance and to optimize RF performance of the modulator. Here, we propose a straightforward procedure which allows determination of  $S_{11}$ ,  $S_{22}$ , and  $\Delta = S_{11}S_{22} - S_{12}S_{21}$  of an arbitrary taper by direct measurements of two test structures. Moreover, the concurrent simulation of a reference taper and measurements on a third test structure allow individual evaluation of  $S_{12}$  and  $S_{21}$ .

#### A. Evaluation of $S_{11}$ , $S_{22}$ , and $\Delta$

Two symmetrical structures are used to evaluate  $S_{11}$ ,  $S_{22}$ , and  $\Delta$  of an input/output taper: the test structure THRU is composed of the taper under test (cascade matrix  $T^1$ ) followed by its symmetrical structure (cascade matrix  $\bar{T}^1$ ), the structure LINE contains  $T^1$ , a  $M$ -length CPW (cascade matrix  $T^M$ ), and  $\bar{T}^1$ . Two matrix equations can be written which relate the measured cascade matrix  $T^T$  and  $T^L$  of THRU and LINE structures, respectively, to the taper under test and the CPW cascade matrices

$$T^1 \cdot \bar{T}^1 = T^T \quad (6)$$

$$T^1 \cdot T^M \cdot \bar{T}^1 = T^L \quad (7)$$

The rank of the resulting  $8 \times 3$  linear system in the variables  $S_{11}$ ,  $S_{22}$ , and  $\Delta$  is equal to 3, and the solution is the following:

$$S_{11} = \frac{T_{12}^L - T_{12}^M - T_{22}^M T_{12}^T}{T_{11}^T \cdot T_{22}^M - T_{11}^L} \quad (8)$$

$$S_{22} = \frac{T_{11}^T \cdot T_{12}^M - T_{11}^T \cdot T_{12}^L + T_{11}^L \cdot T_{12}^T}{T_{11}^T \cdot T_{22}^M - T_{11}^L} \quad (9)$$

$$\Delta = \frac{T_{12}^L \cdot T_{12}^T - T_{12}^M \cdot T_{12}^T - T_{22}^M + T_{11}^L \cdot T_{22}^T}{T_{11}^T \cdot T_{22}^M - T_{11}^L} \quad (10)$$

from which the product  $S_{12}S_{21}$  can be derived.

#### B. Evaluation of $S_{12}$ and $S_{21}$

In order to determine individual values of  $S_{12}$  and  $S_{21}$ , measurements have to be performed on a nonsymmetrical test structure composed of a reference matching taper and the taper under test (cascade matrix  $T^H$ ). The reference taper is an arbitrary coplanar structure which shows as the input geometry (central electrode width, gap, and lateral grounds width), the geometry

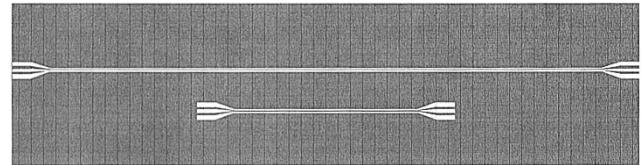


Fig. 1. The 5 mm-and the 15-mm test pattern designed on LiNbO<sub>3</sub> substrate.

of the probe tips, as the output geometry the one of the modulator CPW. Moreover,  $S$ -parameters of the reference structure  $S_{ij}^{ref}$  have to be determined by means of e.m. simulation. The parameters  $S_{12}$  and  $S_{21}$  of the taper under test are then evaluated from the following equations:

$$S_{12} = -\frac{S_{12}^{ref} \cdot \Delta}{T_{22}^H - S_{11}^{ref} \cdot T_{12}^H} \quad (11)$$

$$S_{21} = -S_{21}^{ref} \cdot (\Delta \cdot T_{11}^H + T_{12}^H \cdot S_{22}) \quad (12)$$

### IV. PROCEDURE VALIDATION

Validation of the proposed procedure has been performed by using two structures manufactured on LiNbO<sub>3</sub> substrate and shown in Fig. 1. In both the structures, a coplanar microstrip (with 5 and 15-mm length, respectively) is connected by means of tapered transitions with 80- $\mu$ m pads and 80- $\mu$ m gap.

A six-mode simulation of the transition taper cascaded by a 2.5-mm coplanar line has been performed with ANSOFT HFSS software tool in order to provide the  $S$ -parameters  $S_{ij}^{ref}$  of the reference structure and the characteristic impedance  $Z_c$  of the coplanar line: five higher-order mode have been considered in order to take into account of the effects of taper-line discontinuity. However, it has been verified that the effects of higher-order modes can be neglected, being identical the six-mode and the fundamental-mode  $S$ -parameters. A 31- $\Omega$  value has been found for  $Z_c$ .

The following structures have been used to perform the extraction procedure. The 5-mm and the 15-mm test patterns have been considered as the THRU and the LINE structures to determine  $\alpha$  and  $\beta$  of the coplanar line; the same test patterns have been considered also as the THRU and LINE symmetrical structures needed to determine  $S_{11}$ ,  $S_{22}$ , and  $\Delta$  of the input/output taper; the taper under test is composed of the transition taper cascaded by a 2.5-mm line and an  $M = 10$ -mm line (evaluated starting from  $\alpha$ ,  $\beta$ , and  $Z_c$ ) has to be considered in the LINE structure. The reference taper is composed of the fundamental-mode transition taper simulated with HFSS software tool.

As the taper under test and the reference taper have identical geometric configurations, the 5-mm measured test pattern has been considered as the nonsymmetrical test structure.

The procedure described in Sections II and III has been performed to evaluate  $\alpha$  and  $\beta$  of the coplanar line, and the  $S$ -parameters of the taper under test; in Fig. 2, the  $S$ -parameters of the taper under test are compared to the  $S$ -parameters of the simulated taper (dot line). Moreover, in Fig. 3, the comparison between simulated and measured (dot line) 15-mm test pattern is shown.

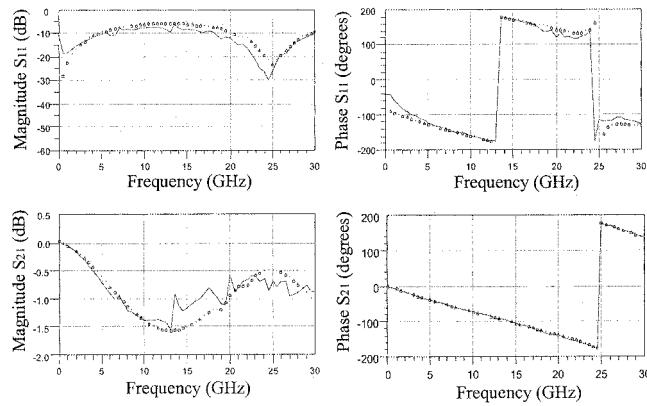


Fig. 2. Comparison between simulated and extracted  $S$ -parameters of the single reference taper.

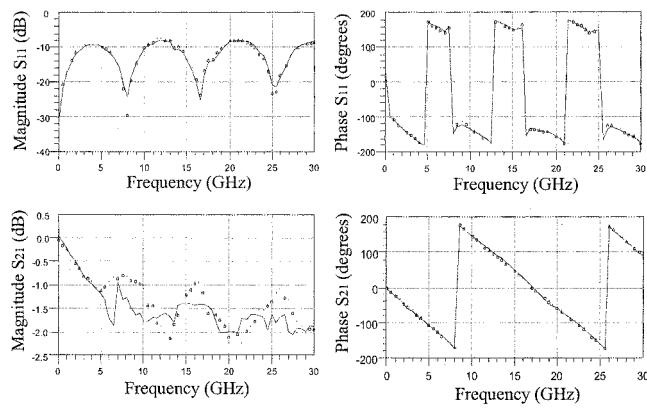


Fig. 3. Comparison between simulated and measured  $S$ -parameters of the 15-mm test patterns.

## V. CONCLUSION

A procedure to determine the electrical  $S$ -parameters of the CPW and of the input and output matching tapers of a Mach-Zehnder electrooptic modulator has been presented. The procedure makes use as input data of both  $S$ -parameters and e.m. simulations of a set of specified test structure. The proposed methodology produces a block partition of the modu-

lator which allows to separate the region where the electrooptic interaction comes (active region) from the input and output tapers used to feed and load the active region. Such a block decomposition is useful during the design of the modulator as well as to develop accurate CAD-compatible modulator models. Procedure validation has been successfully performed by means of both e.m. simulations and measurements performed on proper test structures fabricated on a  $\text{LiNbO}_3$  mask.

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